Name ________________________________

Probability Worksheet

1. If $P(A) = .6$, $P(B) = .5$ and $P(A \cup B) = .9$, find $P(A \cap B)$ and $P(A|B)$.

2. If $P(A) = .2$, $P(B) = .4$ and $P(A \cup B) = .5$, find $P(A \cap B)$ and $P(B|A)$.

3. Given that $P(A) = .3$, $P(B) = .6$, and $P(B|A) = .4$, find the following:
   a) $P(A \cap B)$
   b) $P(A \cup B)$
   c) $P(A|B)$
   d) $P(A^c \cap B^c)$

4. Given that $P(A) = .4$, $P(B) = .5$, and $P(B|A) = .8$, find the following:
   a) $P(A \cap B)$
   b) $P(A \cup B)$
   c) $P(A|B)$
   d) $P(A^c \cap B^c)$

5. Let A and B be two mutually exclusive events for which $P(A) = .25$ and $P(B) = .6$. Find the following:
   a) $P(A \cap B)$
   b) $P(A \cup B)$
   c) $P(A|B)$

6. Let A and B be two independent events for which $P(A) = .25$ and $P(B) = .6$. Find the following:
   a) $P(A|B)$
   b) $P(B|A)$
   c) $P(A \cap B)$
   d) $P(A \cup B)$

7. Let A and B be two disjoint events for which $P(A) = .15$ and $P(B) = .4$. Find the following:
   a) $P(A \cap B)$
   b) $P(A \cup B)$
   c) $P(B|A)$

8. Let A and B be two independent events for which $P(A) = .15$ and $P(B) = .4$. Find the following:
   a) $P(A|B)$
   b) $P(B|A)$
   c) $P(A \cap B)$
   d) $P(A \cup B)$

9. An aerospace company has submitted bids on two separate federal government defense contracts, A and B. The company feels that it has a 60% chance of winning contract A and 30% chance of winning contract B. Given that it wins contract B, the company believes that it has an 80% chance of winning contract A.

a) What is the probability that the company will win both contracts?

b) What is the probability that the company will win at least one of the two contracts?

c) If the company wins contract B, what is the probability that it will not win contract A?

10. Suppose that the aerospace company in #9 feels that it has a 50% chance of winning contract A and a 40% chance of winning contract B. Furthermore, it believes that winning contract A is independent of winning contract B.

a) What is the probability that the company will win both contracts?

b) What is the probability that the company will win at least one of the two contracts?

11. A sporting goods store estimates that 20% of the students at a nearby university ski downhill and 15% ski cross-country. Of those who ski downhill, 40% also ski cross-country.

a) What percentage of these students ski both downhill and cross-country?

b) What percentage of the students do not ski at all?

12. A union's executive conducted a survey of its members to determine what the members felt were the important issues to be discussed during upcoming negotiations with management. Results showed that 74% felt that job security was an important issue, while 65% felt that pension benefits were an important issue. Of those who felt that pension benefits were an important issue, 60% also felt that job security was an important issue.

a) What percentage of the members felt that both job security and pension benefits were important?

b) What percentage of the members felt that at least one of these two issues was important?

13. Let $P(A) = .6$, $P(B|A) = .2$ and $P(B|A^c) = .3$. Use a probability tree to find $P(A|B)$ and $P(A|B^c)$.

14. Let $P(A) = .3$, $P(B^c|A) = .1$ and $P(B|A^c) = .2$. Use a probability tree to find $P(A|B)$ and $P(A|B^c)$.

15. Let $P(A) = .2$, $P(B|A) = .4$ and $P(B^c|A^c) = .3$. Use a probability tree to find $P(A|B)$ and $P(A|B^c)$.

16. Let $P(A^c) = .4$, $P(B^c|A) = .5$ and $P(B^c|A^c) = .6$. Use a probability tree to find $P(A^c|B)$ and $P(A^c|B^c)$.

17. Due to turnover and absenteeism at an assembly plant, 20% of the items are assembled by inexperienced employees. Management has determined that customers return 12% of the items assembled by inexperienced employees, whereas only 3% of the items assembled by experienced employees are returned. Given that an item has been returned, what is the probability that it was assembled by an inexperienced employee?

18. A consumer goods company recruits several graduating students from universities each year. Concerned about the high cost of training new employee the company instituted a review of attrition among new recruits. Over five years, 30% of the new recruits came from the local university, and the balance came from more distant schools. Of the new recruits, 20% of those who were students at the local university resigned within two years, while 45% of other students did. Given that a student resigned within two years, what is the probability that she was hired from the local university? From a more distant university?

19. The vice-president of a hardware wholesaler has asked the bookkeeper to call all customers five days before their account payments are due, as a means of reducing the number of late payments. As a result of time constraints, however, only 60% of customers receive such a call from the bookkeeper. 90% of the customers called pay on time, while only 50% of those not called pay on time. The company has just received a payment on time from a customer. What is the probability that the bookkeeper called this customer?

20. An adhesive technologist concludes from laboratory results that a bond from a new adhesive fails 3% of the time if oil is present on the bonding surface, 12% of the time if the surface is too smooth, and 6% of the time if grit is present on the bonding surface. The bond never fails if the surface is clean. For a particular application by an industrial customer, it is estimated that there is a 5% chance that oil will be on the surface, a 1% chance that the surface will be too smooth, and a .5% chance that grit will be present. This customer has decided to use the new adhesive.

a) What is the probability that the bond will be successful?

b) If the bond is later reported to have failed, what is the probability that grit was present on the bonding surface?

21. When a test is conducted to determine whether or not someone is infected with a particular virus, an incorrect test result can occur in two ways: an infected person may test negative, or a noninfected person may test positive. The latter is called a false positive test. It has been pointed out that the social consequences of false positive tests for the AIDS virus are particularly serious.* Such a false positive test will unnecessarily "stigmatize and frighten many healthy people, because most people consider a positive AIDS test to be a sentence to ghastly suffering and death." Meyer and Paulker therefore assert that it is important that a patient who tests positive for the AIDS virus have a high probability of really being infected. In order to focus on the false positive rate of the test, assume throughout this question that we are dealing with a test that properly identifies all persons who really are infected with the AIDS virus.

a) Assume that 5% of a population to be tested for the AIDS virus really is infected and that the test has a false positive rate of .5%. Find the probability that a person who tests positive really is infected.

b) Would your answer to part a) be higher of lower if more than 5% of the population to be tested were actually infected? Answer the question by referring to the formula for conditional probability without performing any calculations.

c) Assume now that a low risk population is to be tested. Specifically, assume that .01% of the population to be tested is actually infected with the AIDS virus and that the test has a false positive rate of .005% (which is unusually low). Find the probability that a person who tests positive really is infected.

d) What would your answer to c) be if you assumed a false positive rate of .5%?

e) Summarize the implications of your findings in parts a) through d).

*Klemens Meyer and Stephen Paulker, "Screening for HIV: Can we afford the False Positive Rate?" *New England Journal of Medicine (1987):238-41.*